New derivation of a third post-Newtonian equation of motion for relativistic compact binaries without ambiguity

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A third post-Newtonian (3 PN) equation of motion for an inspiralling binary consisting of two spherical compact stars with strong internal gravity is derived under harmonic coordinate condition using the strong field point particle limit. The equation of motion is complete in a sense that it is Lorentz invariant in the post-Newtonian perturbative sense, admits conserved energy of the orbital motion, and is unambiguous, that is, with no undetermined coefficient. In this paper, we show explicit expressions of the 3 PN equation of motion and an energy of the binary orbital motion in case of the circular orbit (neglecting the 2.5 PN radiation reaction effect) and in the center of the mass frame. It is argued that the 3 PN equation of motion we obtained is physically unambiguous. Full details will be reported elsewhere.

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Renewed attention has been paid to a high order post-Newtonian equation of motion governing inspiralling compact binaries in the context of the efforts for direct detection of gravitational waves [1,2]. It is well-known that detectability of the gravitational waves emitted by the binaries and quality of measurements of astrophysical information (e.g. masses) depend on accuracy of theoretical knowledge of the waveforms [1], and hence partly of dynamics of the binaries.

The 3 PN approximation has been a subject of much discussion because of its ambiguity reported originally in Jaranowski and Schäfer [3]. In fact, the 3 PN ADM Hamiltonian in the ADM-type gauge obtained in [3] has two undetermined coefficients ($\omega_{\rm kinetic}$ and $\omega_{\rm static}$) and the 3 PN equation of motion in the harmonic gauge derived by Blanchet and Faye [4] has one coefficient λ undetermined within their framework. Both groups have used Dirac delta distributions, which cause divergences in general relativity, to express the point particles and inevitably they have resorted to mathematical regularizations. Damour *et al.* [5] pointed out that the undetermined coefficients may arise due to unsatisfactory features of the regularizations they have used in [3,4]. Indeed, using the dimensional regularization, the work [5] have succeeded in determining both of the coefficients, namely, $\omega_{\rm static} = 0$, which means $\lambda = -1987/3080$ via relationship established in [6]. ($\omega_{\rm kinetic}$ is related with Lorentz invariance and was fixed in [5,7]. Blanchet and Faye have developed a Lorentz invariant Hadamard Patie Finie regularization [8,9] and do not have any ambiguity other than λ.)

In gravitational wave data analysis, the reduction of predictability of the equation of motion due to the undetermined coefficient can become a problem. In fact, the 3.5 PN phase evolution equation and luminosity [10] unfortunately have four undetermined coefficients, one of which is λ.

Theoretically, a use of Dirac delta distributions and inevitable regularization should be verified in some manner. The perfect (physical) agreements among the results obtained by various authors with various methods [11–13] give a direct theoretical confirmation of the 2.5 PN result first derived by Damour and Deruelle [14]. It is important to achieve 3 PN iteration without introducing singular sources to derive unambiguous result and support the previous 3 PN works which have used Dirac delta distributions.

Based on our previous papers [12,15], we derive a 3 PN equation of motion for two spherical compact stars in harmonic gauge without introducing singular sources. Instead, we apply the strong field point particle limit [16] to deal with strong internal gravity of the stars. Our derivation is satisfactory in a sense that the equation admits conserved energy, is Lorentz invariant, and is unambiguous. In this paper, we shall show both of the 3 PN equation of motion and an associated 3 PN energy of the orbital motion in the center of mass frame and in the case of circular orbit.

Below, we shall explain briefly our yet another derivation of a 3 PN equation of motion. Since this method is different from others, we mention some details specific to our method at the 3 PN order. After deriving an invariant

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energy of the binary orbital motion, we shall compare it with that derived by Blanchet and Faye and fix the λ parameter. Full explanation of our method including computational details will be reported in [17]. See also [12,15].

We write explicitly the post-Newtonian expansion parameter, ϵ, which represents the smallness of the orbital velocities of the stars. The mass scales as ϵ^2 from the post-Newtonian scaling. Then the *strong field point particle limit* [16] is achieved by setting the radius of the star to scale at the same rate as its mass, ϵ^2. The scalings of the mass and the radius enable us to incorporate in the post-Newtonian limit ($\epsilon \to 0$) a limit of a regular point particle whose internal gravity, $\sim$ the mass over the radius, is strong irrelevantly to ϵ.

We derive an equation of motion via surface integrals of the gravitational energy momentum flux going through a sphere surrounding the star. For this method, we prepare two spheres $B_A(\tau) \equiv \{x^k | |\vec{x} - \vec{z}_A(\tau)| \le \epsilon R_A\}$ (A labels the two stars) on the $\tau =$ constant surface, where τ is the time coordinate in the near zone. B_A, called the body zone, is centered at the star A's representative point $z_A^i(\tau)$ and has a radius ϵR_A where R_A is an arbitrary constant and smaller than the orbital separation but larger than the radii of the stars. We make the body zone radius shrink proportionally to ϵ in the near zone coordinate (τ, x^i) to ensure that the field on the body zone boundary due to the star is obtained by multipole expansion when the ϵ zero limit is taken.

The l-th multipole moments of the star A denoted by $I_A^L(\tau)$, including its mass, are defined as volume integrals over B_A of $\Lambda^{\tau\tau} \equiv -g(T^{\tau\tau} + t_{LL}^{\tau\tau}) + \chi^{\tau\tau\alpha\beta}{}_{,\alpha\beta}$ where $g, T^{\mu\nu}$ and $t_{LL}^{\mu\nu}$ are the determinant of the metric $g_{\mu\nu}$, the matter stress energy tensor and the Landau-Lifshitz pseudo tensor. $\chi^{\mu\nu\alpha\beta}{}_{,\alpha\beta}$ arises since we use the wave operator of the flat spacetime instead of that of the curved spacetime when we solve the harmonically relaxed Einstein equation.

$$I_A^L \equiv \epsilon^2 \int_{B_A(\tau)} d^3\alpha_A \Lambda^{\tau\tau} \alpha_A^L, \tag{1}$$

where we introduced the body zone coordinate $\alpha_A^i = \epsilon^{-2}(x^i - z_A^i(\tau))$ and multi-indices notation $L = i_1 \cdots i_l$ ($l \ge 0$: integer). $\alpha_A^L = \alpha_A^{i_1} \cdots \alpha_A^{i_l}$. The mass $m_A \equiv \lim_{\epsilon \to 0} P_A^\tau$ with $P_A^\tau \equiv I_A^0$ so defined would be the ADM mass if the companion star were absent and the body zone radius is taken to be infinite in the body zone coordinate.

In deriving a 3 PN equation of motion, it is important in our formalism to notice that the body zone B_A is a sphere in the frame where the star A orbits, but B_A is *not* a sphere in the generalized Fermi frame [20] where the star A is at rest and the effect of the gravitational field due to the companion star is removed (as much as possible) except for, namely, the tidal effect. We define stars to be spherical in the generalized Fermi frame. Now, we define the "intrinsic" multipole moments $\hat{I}_A^L(\hat{\tau})$ on $\hat{\tau} =$ constant surface in the generalized Fermi coordinate $(\hat{\tau}, \hat{x}^i)$ as a volume integral over a sphere $\hat{B}_A(\hat{\tau})$ centered at the same world event and with the same radius ϵR_A as $B_A(\tau)$. For example, the difference between the symmetric tracefree quadrupole moments is defined as $\epsilon^8 \delta I_A^{<ij>} \equiv \epsilon^8 I_A^{<ij>} - \epsilon^8 \hat{I}_A^{<ij>} = \int_{B_A(\tau)} d^3y_A \Lambda^{\tau\tau} y_A^{<i} y^{j>} - \int_{\hat{B}_A(\hat{\tau})} d^3\hat{y}_A \Lambda^{\hat{\tau}\hat{\tau}} \hat{y}_A^{<i} \hat{y}_A^{j>}$ where $< .. >$ denotes symmetric tracefree operation on the indices between the brackets. For a "spherically symmetric" compact stars, the "intrinsic" tracefree quadrupole moment $\hat{I}_A^{<ij>}$ vanishes. Up to the 3 PN iteration of the gravitational field, $\delta I_A^{<ij>}$ arise mainly due to the Lorentz contraction and can be evaluated as a surface integral over ∂B_A.

$$\delta I_A^{<ij>} = \epsilon^{-6} \frac{1}{2} v_A^k v_A^l \oint_{\partial B_A} dS_k y_A^l y_A^{<i} y_A^{j>} \Lambda^{\tau\tau} + O(\epsilon^4) = -\epsilon^2 \frac{4}{5} m_A^3 v_A^{<i} v_A^{j>} + O(\epsilon^4). \tag{2}$$

In our formalism, it is possible to derive the 3 PN field for an isolated star (by taking limit where the mass of the companion star is zero). $\delta I_A^{<ij>}$ is a necessary term to obtain the correct expression of the 3 PN field for such a system. Although other multipole moments defined over B_A possibly hide purely monopole terms, only the quadrupole moment is found to be relevant up to the 3 PN order. Clean extraction of monopole terms from the multipole moments defined in our previous works is a problem at the 3 PN order specific to our formalism. Blanchet and Faye on the other hand elaborate their generalized Hadamard Partie Finie regularization [8] in a Lorentz invariant manner [9], and have properly taken into account of special relativistic kinematic effects including the Lorentz contraction.

Now, let us briefly explain our derivation of the equation of motion. The local conservation law of the total energy gives an evolution equation for four momentum of the star and relationships among the multipole moments, namely, momentum-velocity relation. The last read as $P_A^i = P_A^\tau v_A^i + Q_A^i + \epsilon^2 dD_A^i/d\tau$ where $P_A^i \equiv \epsilon^2 \int_{B_A} d^3\alpha_A \Lambda^{\tau i}$ and P_A^τ are the three momentum and the energy of the star A. $Q_A^i \equiv \epsilon^{-4} \oint_{B_A} dS_k (\Lambda^{\tau k} - v_A^k \Lambda^{\tau\tau}) y_A^i$ arises since the (pseudo-)stress energy momentum of the field extends outside of the star [12]. Q_A^i can be evaluated explicitly and do contribute the 3 PN velocity momentum relation. We can define the representative point $z_A^i(\tau)$ of the star A by specifying the dipole moment of the star, $D_A^i \equiv I_A^i$ (, e.g., $z_A^i(\tau)$ corresponding to a condition $D_A^i = 0$ may be called the center of mass of the star A from an analogy of the Newtonian dynamics). The relationship between the energy P_A^τ and the mass m_A can be obtained by integrating functionally the evolution equation of P_A^τ, which is expressed as surface integrals and can be evaluated explicitly up to the 3 PN order, in the form as $P_A^\tau = m_A[1 + O(\epsilon^2)]$.

Combining the mass energy relation, the momentum velocity relation, and the evolution equation for the four momentum, we obtain the general form of the equation of motion [12];

$$
\begin{aligned}
m_A\frac{dv_A^i}{d\tau} &= -\epsilon^{-4}\oint_{\partial B_A} dS_k\Lambda^{ki} + \epsilon^{-4}v_A^k\oint_{\partial B_A} dS_k\Lambda^{\tau i} \\
&+\epsilon^{-4}v_A^i\left(\oint_{\partial B_A} dS_k\Lambda^{k\tau} - v_A^k\oint_{\partial B_A} dS_k\Lambda^{\tau\tau}\right) \\
&-\frac{dQ_A^i}{d\tau} - \epsilon^2\frac{d^2D_A^i}{d\tau^2} + (m_A - P_A^\tau)\frac{dv_A^i}{d\tau}
\end{aligned} \tag{3}
$$

Note that $\Lambda^{\mu\nu} = (-g)t_{LL}^{\mu\nu} + \chi^{\mu\nu\alpha\beta}{}_{,\alpha\beta}$ on ∂B_A, since ∂B_A is well outside the star by construction of the body zone. The acceleration in the right hand side of Eq. (3) should be understood to be lower order acceleration than in the left hand side. The terms in the right hand side of Eq. (3) are completely expressed as surface integrals over the body zone boundary except for D_A^i to be specified. The surface integral approach enables us to derive an equation of motion irrelevant to the internal structure of the star (Effects of the star's internal structure on the orbital motion such as tidally induced multipole moments appear through the field and hence the integrand $\Lambda^{\mu\nu}$). The scaling of the body zone radius ϵR_A ensures that we have an equation of motion for compact stars.

The field equation coupled to the matter equations mentioned above is the integrated relaxed Einstein equation under harmonic gauge ($h^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{-g}g^{\mu\nu}$ where $\eta^{\mu\nu} = \mathrm{diag}(-\epsilon^2, 1, 1, 1)$. The harmonic condition is then $h^{\mu\nu}{}_{,\nu} = 0$)

$$
h^{\mu\nu}(\tau, x^i) = 4\int_{C(\tau,x^i)} d^3y\frac{\Lambda^{\mu\nu}(\tau - \epsilon|\vec{x} - \vec{y}|, y^k; \epsilon)}{|\vec{x} - \vec{y}|}, \tag{4}
$$

We split the flat light cone $C(\tau, x^i)$ into four parts; two body zones B_A, near zone outside the body zones N/B surrounding the binary and the far zone outside the near zone. For B_A and N/B contributions to the field $h^{\mu\nu}(\tau, x^i)$, we expand the retarded field about the near zone time τ. Then multipole expansion of the star is used to evaluate the two body zone contributions. The N/B contribution is basically evaluated with help of super-potentials (a super-potential here means a particular solution valid in N/B of a Poisson equation with a non-compact support source in N/B). Unfortunately, it was not possible to find all the necessary super-potentials explicitly at the 3 PN order. For integrands in Eq. (4) such that we could not find super-potentials in closed forms, after making the retarded expansion we leave the Poisson integrals unevaluated and substitute the (not-integrated) field into $\Lambda^{\mu\nu}$ in Eq. (3). Then we perform the surface integrals in Eq. (3) (with respect to the spatial variable x^i in Eq. (4)) first and next perform the remaining volume integrals (with respect to the spatial variable y^i in Eq. (4)). In other words, we extract the parts of the field necessary to derive the equation of motion by interchanging the order of the integrations in Eq. (3) and Eq. (4). As a check, we applied this method on the integrands for which the necessary super-potentials can be derived in closed forms, and found that both methods give the same result. Finally, we have dealt with the integration over the far zone using the DIRE method [13,18] and found that it does not contribute to the 3 PN equation of motion [13,19].

Using the method mentioned above, we obtain a 3 PN equation of motion for a two spherical compact stars binary. We here present the 3 PN relative acceleration in the case of the circular orbit and in the center of mass frame, which is an appropriate equation to inspiralling binaries.

$$
\frac{dV^i}{d\tau} = -\Omega_{\rm ln}^2 r_{12}^i + \epsilon^5{}_{2.5\rm PN}A^i, \tag{5}
$$

where $V^i = v_1^i - v_2^i$ is the relative velocity and ${}_{2.5\rm PN}A^i$ is the relative acceleration at the 2.5 PN order (the radiation reaction term). The 3 PN orbital angular frequency $\Omega_{\rm ln}$ is, (for comparison, we adopt similar notations as in [4])

$$
\begin{aligned}
m^2\Omega_{\rm ln}^2 &= \gamma^3\left[1 + \epsilon^2\gamma(-3+\nu) + \epsilon^4\gamma^2\left(6 + \frac{41}{4}\nu + \nu^2\right)\right. \\
&\left.+\epsilon^6\gamma^3\left(-10 + \nu\left\{-\frac{2375}{24} + \frac{41\pi^2}{64} + 22\ln\left(\frac{r_{12}}{R_0}\right)\right\} + \frac{19}{2}\nu^2 + \nu^3\right)\right] + O(\epsilon^7),
\end{aligned} \tag{6}
$$

where $m = m_1 + m_2$, $\nu = m_1m_2/m^2$, $\gamma = m/r_{12}$, and $\ln R_0 = (m_1\ln(\epsilon R_1) + m_2\ln(\epsilon R_2))/m$. In Eq. (5) with (6), the representative point of the star A, $z_A^i(\tau)$, is defined by setting $D_A^i = \epsilon^4\delta_{AQ}^i = -86\epsilon^4 m_A^3{}_N a_A^i/9$, where ${}_N a_A^i$ is the Newtonian acceleration. This choice makes the three momentum P_A^i parallel to the velocity v_A^i. We note that there is no arbitrary parameter other than the body zone radii ϵR_A in the 3 PN relative acceleration. We here note that it is not allowed to fix the λ parameter by comparing Eq. (6) with the corresponding result of Blanchet and Faye [4],

since the harmonic condition both groups have used does not fix the gauge completely [17] and the expression of the 3 PN orbital angular frequency in terms of the coordinate distance m/r_{12}, Eq. (6), is gauge dependent. (From the same reason, we can not fix λ using Eq. (13) below.)

We can remove away ϵR_A dependence from the 3 PN relative acceleration, Eq. (5), physically by a suitable redefinition of the representative points of the stars. In fact, by setting

$$D^i_{A,\mathrm{New}} = \epsilon^4 \delta^i_{AQ} - \epsilon^4 \frac{22}{3} m^3_{A\,N} a^i_A \ln\left(\frac{r_{12}}{\epsilon R_A}\right) \tag{7}$$

we obtain the 3 PN relative acceleration free from any arbitrary parameter

$$\frac{dV^i}{d\tau} = -\Omega^2 r^i_{12} + \epsilon^5{}_{2.5\mathrm{PN}}A^i, \tag{8}$$

with $m^2\Omega^2 = m^2\Omega^2_{\ln} - 22\epsilon^6\gamma^6\nu\ln(r_{12}/R_0)$. This observation in fact is the case in general cases (i.e., in general orbits *not* in the center of mass frame); The 3 PN equation of motion in general cases we have derived is physically free from any ambiguity.

A reason why we are concerned with $\ln\epsilon R_A$ dependence is the following. Blanchet and Faye have introduced four arbitrary parameters in their regularization procedure, two of which appear in the regularization of the field having two singular points, and the others appear in the regularization of equations of motion for those two points. They showed that the two of those parameters can be gauged away, while the other two were consumed to make their equations of motion conservative (modulo the radiation reaction effect), and they found there remained one and only one parameter, λ, although relationship between energy conservation and regularization parameters associated with point particle description is not clear. Our redefinition of the representative points (7) corresponds to their gauge transformation. Then, their observation makes us check if it is physically allowed to remove the $\ln\epsilon R_A$ dependence in our 3 PN equation of motion, since we introduced only two arbitrary parameters ϵR_A and we have no freedom to make our equation motion conservative by adjusting these two parameters if we remove them away. Thus, we have two problems to be solved in our method; removal of $\ln\epsilon R_A$ and an energy conservation. For lack of space, here we show some facts which support naturality of Eq. (7). The energy conservation problem will be addressed in [17], there we shall show our equation of motion and an associated conserved energy of the binary orbital motion in general cases.

Let us consider the harmonic condition.

$$0 = h^{\tau\mu}{}_{,\mu} = 4\epsilon^4 \sum_{A=1,2}\left[\frac{1}{r_A}\frac{dP^\tau_A}{d\tau} + \frac{r^i_A}{r^3_A}\left(P^\tau_A v^i_A + \epsilon^2\frac{dD^i_A}{d\tau} - P^i_A\right)\right.$$
$$\left. + \sum_{A=1,2}\oint_{\partial B_A}\frac{dS_i}{|\vec{x}-\vec{y}|}\left(\Lambda^{\tau i} - v^i_A\Lambda^{\tau\tau}\right) + \cdots\right], \tag{9}$$

$$0 = h^{i\mu}{}_{,\mu} = \left[\epsilon^4\sum_{A=1,2}\frac{1}{r_A}\frac{dP^i_A}{d\tau} + \sum_{A=1,2}\oint_{\partial B_A}\frac{dS_j}{|\vec{x}-\vec{y}|}\left(\Lambda^{ij} - v^j_A\Lambda^{\tau i}\right) + \cdots\right], \tag{10}$$

where "$\cdots$" are irrelevant terms. These equations are manifestation of the fact that the harmonic condition is consistent with the evolution equation of P^τ_A, the momentum-velocity relation, and the equation of motion (and relations among higher multipole moments, hidden in "$\cdots$"). Thus, if logarithmic dependence of ϵR_A arises from the equation of motion (essentially the second term of Eq. (10)), P^i_A must have the same logarithmic dependence (times minus sign) to ensure harmonicity. This and the momentum velocity relation in turn mean P^τ_A, $v^i_A = dz^i_A/d\tau$ or D^i_A have corresponding logarithmic dependence. We found that P^τ_A have no logarithm up to the 3 PN order. Therefore z^i_A or D^i_A should have logarithms. This is consistent with the fact that a choice of D^i_A determines z^i_A. z^i_A depends on logarithms if the old choice is taken, while it does not if our new choice is taken.

The second fact which supports our interpretation is as follows. We find that the near zone dipole moment D^i_N defined by a volume integral of $\Lambda^{\tau\tau}y^i$ becomes

$$\epsilon^2 D^i_N \equiv \epsilon^{-4}\int_N d^3y\Lambda^{\tau\tau}y^i = \sum_{A=1,2}P^\tau_A z^i_A + \epsilon^2\sum_{A=1,2}D^i_A + \epsilon^{-4}\int_{N/B}d^3y\Lambda^{\tau\tau}y^i. \tag{11}$$

Then if we take the old choice of D^i_A, the volume integral becomes

$$\int_{N/B} d^3y \Lambda^{\tau\tau} y^i = \epsilon^4 \frac{22}{3} \sum_{A=1,2} m_{A\,N}^3 a_A^i \ln\left(\frac{r_{12}}{\epsilon R_A}\right) + \cdots , \tag{12}$$

where terms denoted by "$\cdots$" are independent of R_A. Notice that the near zone dipole moment can be freely determined, say, $D_N^i = 0$, since we can define the origin of near zone freely in general [21]. By taking temporal derivatives of D_N^i twice, we see that $D_{A\mathrm{New}}^i$ gives the definition of $z_A^i(\tau)$ in terms of which the 3 PN equation of motion is independent of ϵR_A.

Finally, we show the 3 PN conserved energy (neglecting the 2.5 PN radiation reaction force) of the circular orbital motion in the center of mass frame. Using Eq. (5), we have

$$\begin{aligned} E_{\ln}(\gamma) = -\frac{m\nu\gamma}{2} \Bigg[& 1 + \epsilon^2 \gamma \left(-\frac{7}{4} + \frac{\nu}{4}\right) + \epsilon^4 \gamma^2 \left(-\frac{7}{8} + \frac{49}{8}\nu + \frac{1}{8}\nu^2\right) \\ & + \epsilon^6 \gamma^3 \left(-\frac{235}{64} + \frac{27}{32}\nu^2 + \frac{5}{64}\nu^3 + \nu \left\{ \frac{10141}{576} - \frac{123\pi^2}{64} + \frac{22}{3} \ln\left(\frac{r_{12}}{R_0}\right) \right\}\right)\Bigg] + O(\epsilon^7) \end{aligned} \tag{13}$$

In terms of $x = (m\Omega_{\ln})^{2/3}$ we obtain the 3 PN energy in an invariant form

$$\begin{aligned} E_{\ln}(x) = -\frac{m\nu x}{2} \Bigg[& 1 + \epsilon^2 \left(-\frac{3}{4} - \frac{1}{12}\nu\right) x + \epsilon^4 \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{1}{24}\nu^2\right) x^2 \\ & + \epsilon^6 \left(-\frac{675}{64} + \left\{\frac{34445}{576} - \frac{205\pi^2}{96}\right\}\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right) x^3 \Bigg] + O(\epsilon^7). \end{aligned} \tag{14}$$

Similarly, using Eq. (8), we have $E(\gamma) = E_{\ln}(\gamma) + (11/3)\epsilon^6 m\nu^2\gamma^4 \ln(r_{12}/R_0)$. Here we note that the difference between $E(\gamma)$ and $E_{\ln}(\gamma)$ is merely due to the redefinition of the dipole moments (or equivalently, a coordinate transformation under the harmonic coordinate condition). The invariant energy $E(x)$ is the same as $E_{\ln}(x)$ but with $x = (m\Omega_{\ln})^{2/3}$ replaced with $x = (m\Omega)^{2/3}$. This third fact that the energy has the same form for both definitions of the representative points of the stars when we write the energy in terms of the orbital angular frequency which is an observable supports that the apparent body zone radii dependence of the 3 PN relative acceleration has no physical effect on the orbital motion.

We have thus derived a 3 PN equation of motion which takes account of strong internal gravity and avoids any ambiguity. Comparing our result, Eq. (14), with the corresponding result in [4], we determine the coefficient undetermined in the Blanchet and Faye 3 PN equation of motion as $\lambda = -1987/3080$. This value of λ is consistent with the result of Damour, Jaranowski, and Schäfer [5]. Thus, our result (indirectly) validates their use of the dimensional regularization in the ADM Hamiltonian approach in the ADMTT gauge. Finally, we note that Blanchet *et al.* [22] have recently obtained the same value of λ, who computed a 3 PN equation of motion in the harmonic gauge using dimensional regularization.

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